# Gravity-Model Errors in Mobile Inertial-Navigation Systems

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**A high-accuracy inertial-navigation system (INS) is to be used to navigate a land-mobile vehicle travelling at about 30 knots, over time periods of 2-4 h or less. Important sources of navigation error are the errors in modeling the anomalous gravity forces acting on the vehicle. To obtain acceptable performance, the INS must be accurately compensated in real time for the anomalous gravity. A significant reduction in the growth of navigation errors can be obtained by stopping the vehicle periodically and processing zero-velocity updates in an on-board navigation filter. The land-navigation system design considerations that affect the growth of navigation error due to the gravity-model errors are examined here. These include the grid-spacing of the data base used to derive the real-time gravity compensation, the use of odometer and zero-sideslip measurement data for in-transit INS updating, and the frequency and accuracy of the at-rest zero-velocity updates which are the key to high-accuracy land navigation.**

## Introduction

THERE has been considerable interest recently in the use of an inertial-navigation system (INS) for high-accuracy navigation of land-mobile vehicles, e.g., of the order of 0.1 n.mi./h or better. An important reason for this is that one of the methods currently under study for the deployment of small intercontinental ballistic missiles (SICBMs) involves the use of land-mobile vehicles traveling over the operational areas of interest. The self-contained nature of inertial-navigation systems and their immunity to enemy jamming make them extremely attractive as land-mobile vehicle navigation systems. The focus in this paper is on land-mobile vehicle navigation for time periods of a few hours or less. An INS used for land navigation will develop position errors that increase with the travel time of the vehicle. Improving the accuracy of the inertial instruments will control the growth of position errors to some extent. Ultimately, however, the growth of INS errors is limited by gravitational forces acting on the vehicle, which are not modeled correctly in the navigation software.

There are several different ways for coping with the undesirable effects of gravity modeling errors. The simplest way conceptually is to stop the vehicle periodically at presurveyed benchmarks (PBMs) at which the navigation-system position and gravity-model error estimates can be reset and, if necessary, the inertial measurement unit (IMU) can be realigned physically or its alignment-error estimates can be updated. Economic and operational considerations relating to the data-collection process, however, make it desirable to limit the required number of PBMs to as small a value as possible.

A better scheme for controlling navigation-error growth, which has been employed for many years in inertial surveying systems, is through the use of zero-velocity updates (ZUPTs).[1-4] Here the vehicle stops peridically for a few minutes, during which time ZUPTs are processed in an on-board filter to update the estimates of vehicle position, IMU alignment, and other INS errors. The effectiveness of this scheme depends on how well the velocity errors measured by the ZUPTs correlate with the other errors being estimated in the filter. This in turn is a function of the random errors in the INS, the correlation-distance parameter of the gravity-model errors, and the vehicle operational scenario itself.

Commercial inertial surveying systems[1,2] have achieved extremely high accuracies using ZUPTs alone. An important reason for this is that the ZUPTs are typically taken at an extremely high rate, e.g., every 5-10 min or less. In addition, by carefully planning the survey missions to take advantage of repetitive tracks and closed traverses, the system performance is enhanced. The use of post-mission smoothing of the recorded data helps still further.

In the problem under consideration here, an important objective is to minimize the frequency of stops required for ZUPTs during the travel of the vehicle to its destination point. Stopping every 5-10 min or less for ZUPTs, as is done in current inertial surveying systems, is clearly undesirable. Under these conditions, the use of other types of navigation information, such as real-time gravity-model compensation or odometer-derived ground velocity, needs to be considered as a means of obtaining the desired navigational accuracies.

This paper is primarily concerned with the effects of gravity-model errors on the performance of the INS for a land-mobile vehicle. The major emphasis is on the use of ZUPTs to contain INS errors so that the number of stops required at survey points (PBMs) is kept to a minimum. Methods are examined for reducing the sensitivity of INS performance to the gravity-model errors by updating the INS in-transit, using odometer-derived speed and zero-sideslip measurements, and by appropriately controlling the travel scenario of the vehicle.

## Gravity Model Errors

The land-navigation system INS basically solves for vehicle position $r$ and ground velocity $v$ by mechanizing a set of equations equivalent to[5,6]

$$\dot{v} = s + g - (w_{EN} + 2w_{IE}) \times v \quad (1)$$

$$\dot{r} = v - w_{EN} \times r \quad (2)$$

where $s$ is the specific force acting on the vehicle as measured by the INS accelerometers, $w_{IE}$ the Earth's angular rotation rate, and $w_{EN}$ is the computed angular velocity of the navigation frame with respect to the Earth. The navigation frame is the computational frame in which the above equations are implemented. The quantity $g$ is the computed plumb-bob gravity force acting on the vehicle, and contains both mass-attraction and centripetal components.

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The particular quantity of interest here is the gravity-model error ($\delta g$), which is the difference between the true gravity at the point of interest $g_T$ and the computer-modeled gravity $g_c$, i.e.,

$$\delta g = g_T - g_c \tag{3}$$

For navigation-system performance analysis, it is most convenient to utilize statistical representations for the gravity-model error, preferably ones that can be cast in the state-space format of linear differential equations driven by white-noise processes.[11] Many different models have been proposed, ranging from the simple first-order Markov models described in early studies,[12] the second-order Markov models used in subsequent studies,[15,16] and the third-order Markov models most widely used at the present time.[7,8,13,14,17]

For the particular application of interest here, i.e., the deployment of SICBMs from a land-mobile vehicle, an extremely accurate INS is required. Under these conditions, very accurate real-time compensation of gravity is also required. To generate a statistical model for the post-compensation residuals, the techniques developed by Harriman[16] will be used.

The above method assumes a grid of gravity measurement data is available in the region of interest. A statistical spherical-harmonics model such as the one by Tscherning and Rapp,[20] with suitably adjusted coefficients, is used to represent the local field. The residual-model omission errors are then computed, as described in Ref. 16, under the assumption that only those components of anomalous gravity with wavelengths greater than twice the gravity-measurement data grid spacing are compensated for in the model.

The output data from the Harriman model are the power spectral densities of the along-track and cross-track deflection residuals as functions of spherical-harmonic wavenumber. For use in the analysis of INS performance, on the other hand, it is more convenient to work with frequency or time-domain models. If the vehicle speed is constant or slowly varying, then the required conversions are easily implemented.

The result is that the deflection residuals can be reasonably well approximated by passing white-noise processes through second-order shaping filters. Since there is no cross-correlation between the cross-track and along-track gravity-model errors, independent upcoupled processes can be used. The shaping filter transfer function is given by

$$\frac{x_0}{x_i} = \frac{1}{p^2/w_0^2) + (2\zeta p/w_0) + 1} \tag{4}$$

where $x_i$ and $x_0$ are the filter input and output, respectively, $\zeta$ is the filter damping ratio, and $p$ the operation of differentiation with respect to time. The quantity $w_0$ is the undamped natural frequency for the model. The value of $w_0$ is related to the speed of the vehicle ($v$), which is assumed essentially constant, by the relation

$$w_0 = kv/r_E \tag{5}$$

where $k$ is the maximum wavenumber in the compensated disturbance model, and $r_E$ is the radius of the Earth. The spectral density of the white-noise process driving the shaping filter is given by

$$q^* = 4\zeta\sigma_{x0}^2/w_0 \tag{6}$$

where $\sigma_{x0}$ is the rms value of the vertical-deflection residual provided by the second-order filter.

The assumed parameter values for the models of the post-compensation gravity-model errors experienced by the INS are summarized in Table 1. Two different parameter sets are shown, corresponding to medium-accuracy and high-accuracy gravity compensations. These particular models include both the commission and omission errors.[16] The rms values for the commission errors are much less than those for the omission errors.

A set of power-spectral-density (PSD) curves for the post-compensation gravity-model errors based on the above model is given in Fig. 1 for a 5-n.mi. gravity data grid separation and a vehicle speed of 30 knots. The units for the PSDs are $\mu g^2$/rad/s. The PSDs are plotted against frequency, normalized to the undamped INS Schuler frequency ($w_s$), whose period is about 84 min (0.00125 rad/s).

The rms vertical deflections for the particular grid spacing used in Fig. 1 are about 4 mgals (0.8 arcsec) each. The PSDs for the two components, however, as can be seen, are significantly different. It is important to recognize that although the cross-track and along-track deflection PSDs and autocorrelation functions are independent of the vehicle track direction, they are not necessarily equal to each other.[14,16,17] For the particular vehicle speed of 30 knots assumed in Fig. 1, the model undamped natural frequency ($w_0$) is about 0.0053 rad/s, which is about a factor of 4 higher than the INS resonant frequency. Decreasing the vehicle speed to about 7-8

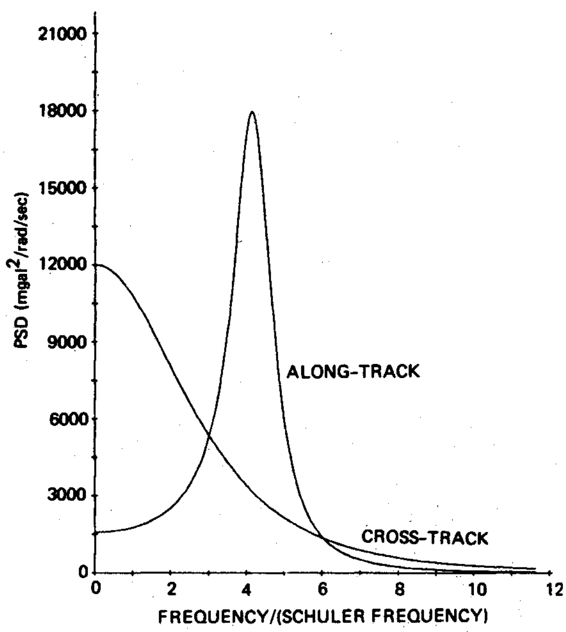



**Fig. 1 PSDs of post-compensation gravity-model errors experienced by INS moving at 30 knots.**

**Table 1 Gravity-model-error parameters[a]**

| Quality of compensation | Data grid spacing | Maximum wavenumber | rms deflections, $\mu g$ Along track | Cross track | Damping ratio Along track | Cross track |
|---|---|---|---|---|---|---|
| High accuracy | 5 n.mi. | 2200 | 3.8 | 4.0 | 0.15 | 1.0 |
| Modest accuracy | 15 n.mi. | 700 | 18.2 | 19.1 | 0.15 | 1.0 |

[a]Model based on Harriman method[16]; second-order shaping filters used to generate frequency and time domain models; models represent post-compensation residuals.

Table 2 Study-model IMU

| Error source | rms value |
|---|---|
| Gyro drift bias | 0.0015 d/h |
| Random-walk angle | 0.0005 d/$\sqrt{h}$ |
| Random-walk rate | 0.0001 d/h·/$\sqrt{h}$ |
| Accelerometer bias | 10 $\mu$gs |
| Accelerometer scale factor | 10 ppm |
| Accelerometer random walk | 1 $\mu g/\sqrt{h}$ |
| Azimuth alignment | 3 mrad |
| Level alignment | 1 mrad/axis |

knots in the above model would place the undamped natural frequency ($w_0$) for the deflections close to the INS resonant frequency ($w_s$). This could lead to an increased sensitivity of INS errors to gravity-model errors.

## INS Performance Model

A relatively simple IMU performance model is utilized in the study of this paper. A gimballed IMU is assumed, with a psi-angle formulation[5,19] of the standard INS error equations. Included as state elements in the dynamics model are the errors in vehicle position, vehicle velocity, IMU alignment, gyro drift bias, accelerometer bias, and accelerometer scale factor. Other IMU errors modeled are the gyro random-walk errors in both angle and rate, along with a random-walk error in acceleration. The gravity-model errors driving the INS, as noted earlier, are represented by the Harriman model.[16]

The performance-model parameters for the assumed INS are given in Table 2. The model is intended to be a generic model for a high-accuracy INS. The indicated numbers were arbitrarily chosen and do not reflect any particular set of gyros and accelerometers.

## Zero-Velocity Updates

An undamped INS used as a land-navigation system with no external updates will develop position errors that increase with time, due to error sources such as gyro drift bias and gravity-model errors. These effects are discussed in the literature.[5,6,9,10] One technique that has been found to be extremely effective in controlling the growth of position errors in land-based inertial surveying systems is the use of zero-velocity updates or ZUPTs. Detailed discussions of the use of ZUPTs for inertial survey systems can be found in several publications.[1-4]

The use of ZUPTs is essential to contain the growth of navigation errors in the mobile inertial-navigation system application of interest here. For this reason, a brief description of ZUPTs and the way they might be used for land-navigation system updating is given next.

The basic ZUPTing concept is extremely simple. The land-mobile vehicle is stopped periodically for a short time interval of 2-3 min or less. During this period, measurements are taken of INS-indicated at-rest ground velocity of the vehicle (ideally zero). These measurements are then processed in a minimum variance filter[11] on board the vehicle to obtain up-to-date estimates of vehicle position, velocity, IMU-alignment errors, and other quantities of interest included as navigation filter states. To facilitate the real-time processing of the INS-indicated velocity data, it is desirable to prefilter segments of data first, then process the smoothed velocity estimates in the on-board filter.

The ability of the ZUPTs to improve navigation-system performance depends primarily on the cross-correlations between the observed velocity errors and the other quantities for which estimates are desired, e.g., vehicle position and IMU-alignment errors. If the dominant INS errors are bias-type errors, then the correlations will tend to be strong, and the ZUPTs will be very effective. If, on the other hand, the primary INS errors are time-varying in nature, e.g., gravity-model or gyro random-walk errors, then the correlations will be weaker and the ZUPTs will be less effective. Decreasing the time period between vehicle stoppings for ZUPTs would, of course, help in the latter case, but this is not a desirable solution.

It is important to recognize that the filter weights used in the processing of the ZUPTs are determined by the statistical models provided for the errors of the INS in the on-board filter. Accurate models are essential to most effectively process the ZUPTs, particularly where the desired estimates are being derived through cross-correlations rather than by direct observations. The studies of this paper assume a smart filter with correct statistical models for all INS and other error sources.

It should also be pointed out that vehicle vibrations sensed by the IMU during the ZUPTing period can introduce significant errors into the ZUPT measurements. To circumvent this problem, the observations must be taken over a sufficiently long time period to smooth out the effects of the vibrations as far as possible. The studies of this paper assume that the raw INS data are prefiltered for a 1-min interval prior to incorporation into the land-navigation filter. The error in the smoothed ZUPTs is assumed to be zero-mean Gaussian noise with an rms value of 0.01 ft/s.

## System Performance Studies

### General Information

To provide more specific insights into the effects of gravity-model errors on mobile inertial navigation systems, the results from some land-navigation studies performed at The Charles Stark Draper Laboratory are presented next. The data to be shown are rms estimation errors computed from linear-covariance simulation runs.

Consistent with the objective of obtaining a basic understanding of the effects of gravity-model errors in a land-mobile INS, an extremely simple mission scenario has been assumed for this study. The vehicle starts at a survey point (PBM) located at a latitude of 45 deg. Vehicle position, velocity, and gravity disturbances are assumed to be known without error at this point. A single-position leveling and gyrocompassing procedure then takes place for a 20-min interval to realign the INS, with the vehicle remaining at the survey point. The inputs to the navigation filter during this period are the preflitered ZUPTs (0.01 ft/s rms error) at 1-min intervals.

The vehicle then travels in an easterly direction to the scenario end point, which is an unsurveyed point located about 120 n.mi. from the starting point. The nominal transit speed for the vehicle is 30 knots (50 ft/s). During the period of travel, the vehicle is permitted to stop periodically to take ZUPTs to be used to update the INS. The vehicle stopping intervals are typically of the order of 2-3 min.

To keep the scope of this study within reasonable bounds, it is assumed that the vertical channel of the INS is operated at all times with essentially no errors present. This implies operationally that the INS is updated continuously in real time, using terrain-elevation information (or its equivalent) from a data base stored in the vehicle.

### IMU Errors Alone

As a starting point, it is useful to examine the performance of the study-model INS for the case where no gravity-model errors are present. The INS performance model is as in Table 2, and the scenario is as described in the preceding section for a vehicle transit speed of 30 knots.

The rms position errors over a 4-h interval with no enroute vehicle stops are shown in Fig. 2. Gyro drift bias and random-walk errors, as expected, contribute to the divergent growth of position errors over the interval of interest. When vehicle stops are permitted, e.g., at 30-min intervals, and ZUPTs are processed to update the INS state vector, then a significant improvement in long-term position-error growth is obtained. This is indicated in Fig. 3. Two factors contribute to the reduced position-error growth: the resetting of the velocity errors at the ZUPT times to small values, and the cross-correlation between the ZUPT-indicated velocity errors and the current INS position errors. Stopping the vehicle more frequently than indicated in Fig. 3, e.g., every 10 min, will further reduce the rate of error growth, but at the expense of extended mission duration.

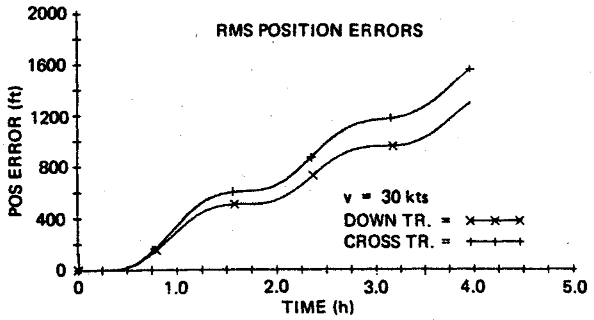



Fig. 2 IMU errors alone, no ZUPTs.

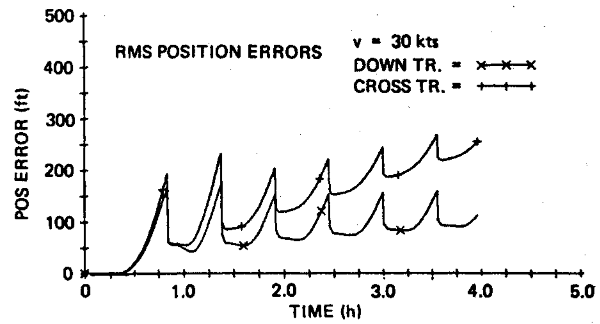
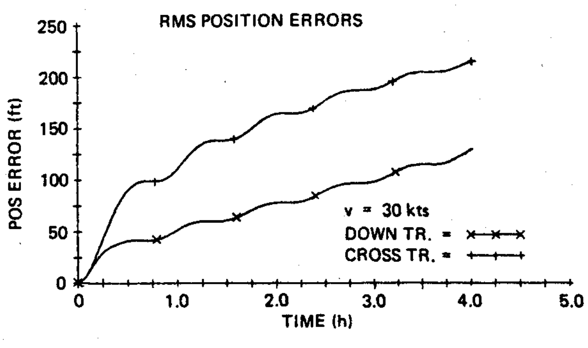
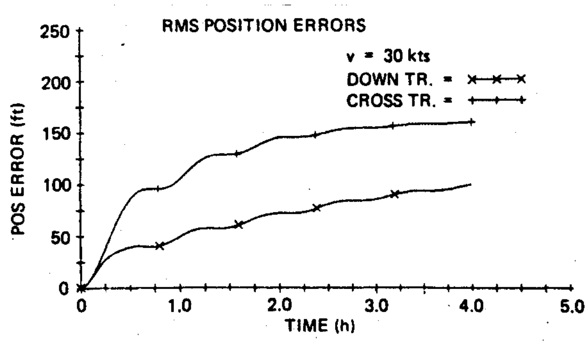



Fig. 3 IMU errors alone, ZUPTs at 30-min intervals.




a) No odometer or zero-sideslip updates.




b) Odometer and zero-sideslip updates (1 fps rms error).

Fig. 4 Gravity-model errors alone, no ZUPTs, high-accuracy compensation.

**Gravity-Model Errors Alone**

The effects of gravity-model errors on INS performance during the land-navigation phase are examined here for the case where the gravity-model error is the only error source driving the INS. All inertial instrument errors are assumed to have zero values. The particular representations used for the gravity-model errors are as indicated in Table 1.

The rms position errors are shown in Fig. 4 for a case where the gravitational forces on the vehicle are compensated very accurately (5-n.mi. data grid). The vehicle transit speed is 30 knots and no ZUPTs are taken. For these particular conditions, the gravity-disturbance model's undamped natural frequency ($w_0$) is 0.0053 rad/s, which is about four times higher than the Schuler frequency ($w_s$) of 0.0012 rad/s of the undamped INS. Under these conditions, it is not surprising that the position-error components of Fig. 4 are unequal, even though the rms deflection components (as in Table 1) are not significantly different. The difference in the low-frequency PSDs [given by Eq. (6)] for the deflection components contribute to the position-error differences.

Decreasing the vehicle transit speed to 10 knots shifts the resonant frequency ($w_0$) of the gravity-model errors closer to the INS Schuler frequency. This results in an increase in the rms position errors, as indicated in Fig. 5. The error components in this case are more nearly equal than in the case of Fig. 4, because the PSDs of the deflection components in the vicinity of the undamped INS Schuler frequency are more nearly equal.

If the gravitational forces are less accurately compensated, then an increase in navigation errors results. This is illustrated in Fig. 6 where gravity data from a 15-n.mi. spaced grid are used to compensate the INS in a land-mobile vehicle travelling nonstop at 30 knots.

The possible benefits from updating the INS in transit with odometer-derived speed and zero-sideslip measurements are considered next. Basically, two velocity-component measurements are utilized here: a forward-velocity measurement derived from the odometer and acting along the longitudinal axis of the vehicle, and an implied zero-velocity measurement parallel to the lateral axis of the vehicle. The zero-lateral-velocity measurement, or zero-sideslip measurement, as it is more commonly called, has been successfully used by Taylor, Pasik, and Fish[18] for updating an INS on board a land-mobile vehicle. Of particular interest in that study[18] is the fact that field-test navigation performance results are presented that show the benefits of this type of measurement. It is important to recognize that the effectiveness of the in-transit velocity updates depends upon the magnitudes and correlation times of the errors in both the gravity model and the velocity-update measurements.

The implementation of the in-transit velocity updates in this study was accomplished by processing the velocity-component measurements in an on-board minimum-variance filter, using

Table 3 Gravity-model-error-alone result summary[a]

| | | rms position errors, ft | |
|---|---|---|---|
| ZUPTs | Damping | Along track | Cross track |
| High-accuracy gravity-model compensation | | | |
| N[b] | N | 130 | 213 |
| N | Y | 100 | 160 |
| Y | N | 37 | 55 |
| Y | Y | 36 | 54 |
| Modest-accuracy gravity-model compensation | | | |
| N | N | 1460 | 1400 |
| N | Y | 432 | 527 |
| Y | N | 104 | 202 |
| Y | Y | 103 | 196 |

[a]Errors are at end of 120-n.mi. trip; average vehicle transit speed of 30 knots; no inertial-instrument errors. [b]N = no, Y = yes.

a 30-s updating interval. The measurement errors for each velocity component were modeled as the combination of a time-correlated first-order Markov process along with an uncorrelated random error. An rms value of 1 ft/s was assumed for the time-correlated error, and an rms value of 1 ft/s for the uncorrelated error. The relatively large time-correlated error in the zero-sideslip measurement is intended to account for the misalignment of the vehicle's rear axle relative to the IMU. Land-navigation performance comparisons are given in Figs. 4 and 6 for cases with high-accuracy and modest-accuracy gravity compensations.

The key results of the in-transit velocity update study are given in Fig. 7, where the rms position errors are shown at the end of a nonstop (no zero velocity updating) trip of 120 n.mi. at a vehicle transit speed of 30 knots. The errors are presented here as a function of the rms value of the time-correlated error in the velocity-update measurements. A correlation time of 3 h is assumed in these data for the time-correlated velocity error. Two sets of data are shown: one for extremely accurate gravity compensation (5-n.mi. grid spacings); the other with moderately accurate compensation (15-n.mi. grid spacings).

The main points to be seen from Fig. 7 are the following. When the gravity compensation is of modest-accuracy (15-n.mi. grid spacings), in-transit updating of the INS with odometer-derived forward velocity and zero-sideslip velocity is extremely useful, even with velocity-component errors of the order of 1-2 ft/s (rms values). The undamped-INS along-track and cross-track rms errors at the end point in this case are about 1460 and 1400 ft, respectively. With an accurately compensated gravity field (5-n.mi. grid spacings), on the other hand, the benefits of the in-transit velocity updates are much

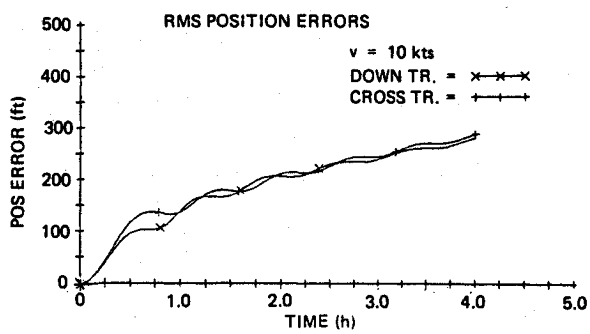



**Fig. 5 Gravity-model errors alone, no ZUPTs high-accuracy compensation, (5-n.mi. grid spacings).**

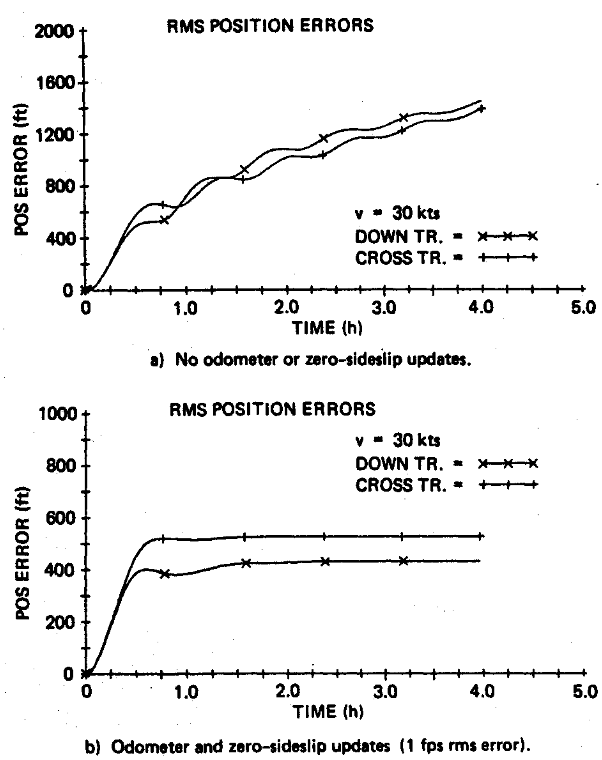



**Fig. 6 Gravity-model errors alone, no ZUPTs, modest-accuracy compensation (15-n.mi. grid spacings).**

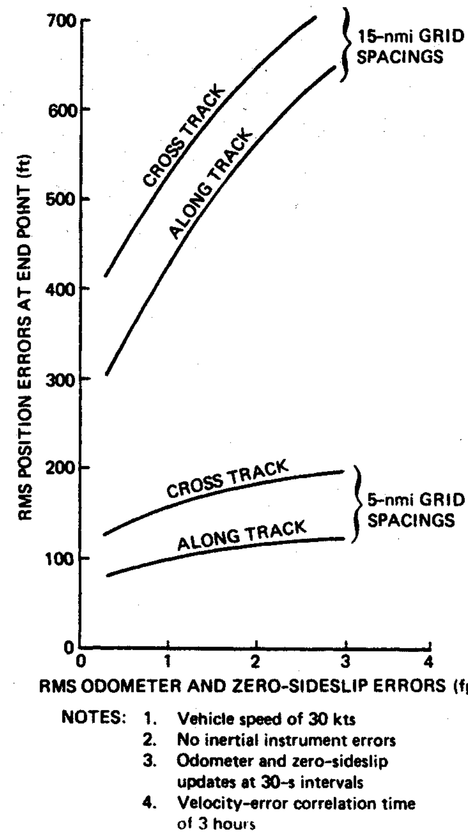



**Fig. 7 Updating the INS with odometer and zero-sideslip data only on non stop trip of 120 n.mi.**

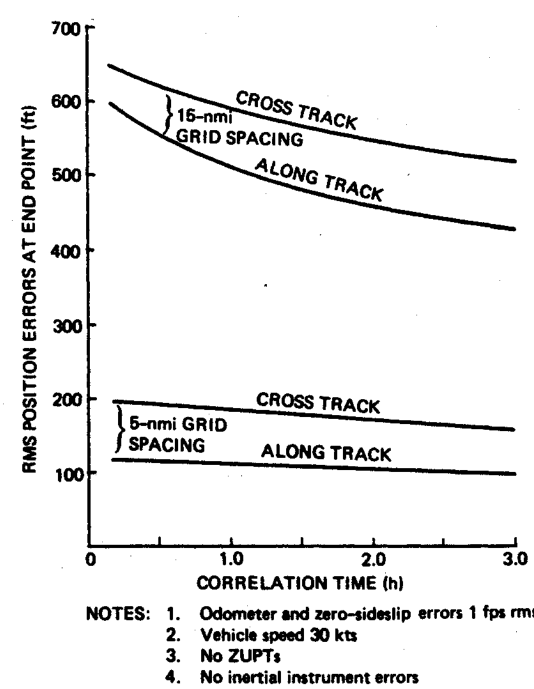
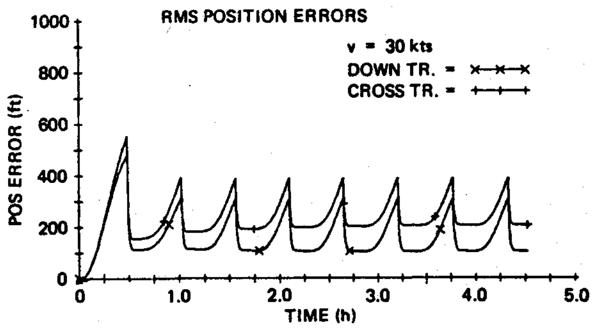
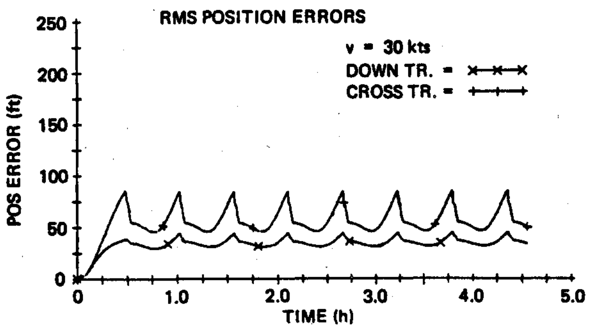



**Fig. 8 Sensitivity of position errors to odometer and zero-sideslip error correlation time.**



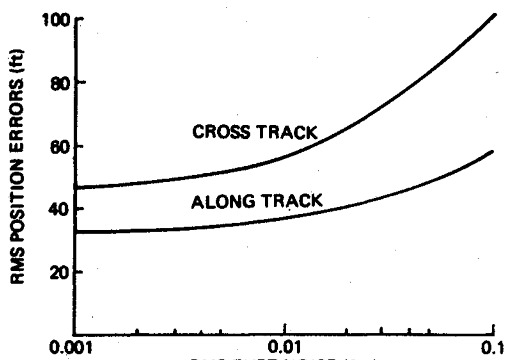

NOTES: 1. Vehicle speed of 30 kts
2. 15-nmi grid spacings for gravity data
3. No odometer or zero-sideslip dates

**Fig. 9 Gravity-model errors alone with medium-accuracy compensation, ZUPTs every 30 min.**




NOTES: 1. Vehicle speed of 30 kts
2. 5-nmi grid spacings for gravity data
3. No odometer or zero-sideslip

**Fig. 10 Gravity-model errors alone with high-accuracy compensation, ZUPTs every 30 min.**

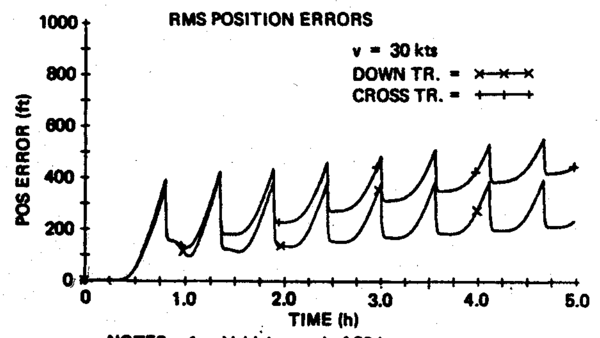



NOTES: 1. Vehicle transit speed of 30 kts
2. Stops for ZUPTs for 3 minutes at 30-minute intervals
3. Gravity model grid spacing of 5-nmi
4. No inertial-instrument errors
5. No odometer or zero-sideslip updates

**Fig. 11 Sensitivity of position errors after 120-n.mi. trip to rms value of ZUPT noise.**

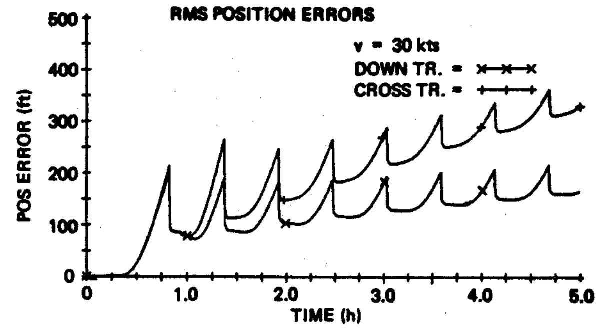
smaller, as can be seen from Fig. 7, even with odometer-speed and zero-sideslip measurement errors greater than 1.0 ft/s. The undamped-INS along-track and cross-track rms errors at the end point in this case are 130 and 213 ft, respectively.

In the data of Fig. 7, a 3-h correlation time was assumed for the in-transit velocity-update errors. The sensitivity of the end-point position errors to this correlation time is shown in Fig. 8 for an rms velocity-update error of 1 ft/s. For the high-accuracy gravity compensation (5-n.mi. grid spacings) the end-point position errors are insensitive to the correlation




NOTES: 1. Vehicle speed of 30 kts
2. ZUPTs for 3 minutes for every 30 minutes
3. Odometer and zero-sideslip updates of INS

**Fig. 12 Gravity-model errors and IMU errors included, modest-accuracy compensation.**




NOTES: 1. Vehicle speed of 30 kts
2. ZUPTs for 3 minutes every 30 minutes
3. Odometer and zero-sideslip updates of INS

**Fig. 13 Gravity-model errors and IMU errors included, high-accuracy compensation.**

time. With the modest-accuracy compensation (15-n.mi. grid spacings), the rms end-point position errors increase somewhat as the correlation time is decreased.

If the land-mobile vehicle stops periodically for ZUPTs, then the rms position error growth as a result of gravity-model errors is significantly reduced. This is shown in Figs. 9 and 10 for cases where the vehicle is traveling at 30 knots, with 3-min stops for ZUPTs every 30 min. The results of Fig. 9 assume medium-accuracy gravity compensation (15-n.mi. grid spacings), whereas those of Fig. 10 assume high-accuracy gravity compensation (5-n.mi. grid spacings). The ZUPTs significantly arrest the growth of INS position errors as a result of gravity-model errors. It should be noted, however, that since the ZUPTs depend on correlations for their updating (not direct measurements), accurate models should be provided in the on-board filter for all important INS error sources.

It should be recognized that decreasing the period between ZUPTs, e.g., to 5-10 min or less, will improve navigation-system performance. In the problem of interest here, however, the desire to minimize the frequency of ZUPTs is of prime importance.

In all the studies thus far, an rms random error of 0.01 ft/s has been assumed for each smoothed ZUPT measurement. The sensitivity of end-point position errors to the rms value for the ZUPT noise is examined in Fig. 11. The important point to be seen is that the sensitivity is small for reductions in the rms values below 0.01 ft/s. Increasing the rms values from 0.01 to 0.1 ft/s does, however, significantly increase the end-point rms position errors. It should be noted, however, that no inertial-instrument errors are included in these data.

A summary of the key results for the studies with gravity-model errors alone driving the INS is given in Table 3, where the rms end-point position errors are shown after a 120-n.mi. trip. The main points from these data are the following. If the gravitational forces on the vehicle are very accurately compensated (5-n.mi. grid spacings), then the use of ZUPTs periodically, e.g., every 30 min, permits extremely small end-point rms position errors (55 ft or less for each component). In-transit velocity updating of the INS does not help much in this case, with the assumed velocity-update errors. If, on the other hand, the gravitational forces are compensated to only a modest accuracy (15-n.mi. grid spacings), then the in-transit velocity updates do provide useful error reductions. If, however, ZUPTs are taken sufficiently often, e.g., every 30 min, then small end-point rms poisiton errors are obtained in this case also (200 ft or less for each component). If ZUPTs are used under the conditions assumed in this study, then in-transit velocity updating of the INS does not provide a significant additional reduction in errors.

### Gravity-Model Errors Combined with IMU Errors

The performance of the land-navigation system is examined here with both the gravity-model errors of Table 1 and the inertial instrument errors of Table 2 driving the inertial navigation system. ZUPTs are taken at 30-min intervals for a period of 3 min to contain the growth of navigation errors. In addition, odometer-derived forward velocity and zero-sideslip measurements are processed in the navigation filter at 30-s intervals to update the INS.

The navigation system position errors are shown in Fig. 12 for the case where the gravitational forces are compensated to a modest accuracy. After the vehicle has travelled 120 n. mi., the rms along-track and cross-track position errors are about 208 and 409 ft, respectively. This is significantly smaller than the 2020- and 2213-ft errors that occurred without zero-velocity updating or in-transit velocity updating of the INS.

When the INS is compensated to high accuracy for the gravitational forces on the vehicle, still smaller end-point position errors are obtained after a trip of 120 n.mi. The rms position errors in this case are shown in Fig. 13. The end-point position errors in this case after a 3-min period of ZUPTs are about 160 and 320 ft, respectively, for both the along-track and cross-track components.

## Conclusions

Gravity-model errors can be significant contributors to the position errors of a land-navigation INS operating over periods of 2-4 h. If operational requirements limit the frequency of vehicle stops for zero-velocity updates to every 30 min or more, then accurate real-time gravity compensation is necessary to limit the rms position errors to about 100 ft or better per component. A gravity data base with a grid spacing of the order of 5 n.mi. is implied here. In-transit updatings of the INS with odometer-derived forward-speed and zero-sideslip lateral-speed measurements help INS performance, but their benefits depend on the magnitude of the velocity-measurement and gravity-model errors. As the accuracy of the in-transit velocity updates is improved or the frequency of at-rest zero-velocity updates is increased, the accuracy requirement for real-time gravity compensation of the INS becomes less severe.

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